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THERMAL CONDUCTION IN CLUSTERS OF GALAXIES

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ABSTRACT

We estimate the thermal conductivity of a weakly collisional magnetized plasma with chaotic magnetic field fluctuations. When the fluctuation spectrum extends over two or more decades in wavevector, we find that thermal conduction is very efficient; the conduction coefficient is only a factor of ∼5 below the classical Spitzer estimate. We suggest that conduction could play a significant role in cooling flows in clusters of galaxies.

Subject headings: conduction — cooling flows — galaxies: clusters: general — magnetic fields

1. INTRODUCTION

Hot X-ray–emitting gas is ubiquitous in clusters of galaxies (Sarazin 1988). Since the X-ray emission is energetically important in many clusters, it is believed that a significant amount of mass must continuously cool and drop out of the intracluster medium (Fabian 1994). The mass deposition rate is estimated to be as much as several hundred $M_\odot$ yr^{-1} in some clusters (see, e.g., David et al. 2001 and Allen et al. 2001).

Direct evidence of the cooling gas has, however, been scarce (Fabian 1994). In particular, recent observations with *XMM-Newton* (Böhringer et al. 2001; Molendi & Pizzolato 2001) and *Chandra* (Fabian et al. 2001) have failed to find the multitemperature gas one expects in a cooling flow. The observations suggest that mass dropout may be less significant than previously thought. A reduced level of mass dropout is possible if there is a source of heat to replace the energy that is lost through X-ray emission, but no clear heat source has yet been identified (Fabian et al. 2001).

The inner region of a cluster (R approximately a few times 10 kpc), where mass dropout seems to be occurring, is typically cooler than the rest of the cluster. Therefore, an often discussed source of heat is thermal conduction from the hot outer regions of the cluster to the center (Binney & Cowie 1981; Tucker & Rosner 1983; Bertschinger & Meiksin 1986; Bregman & David 1988; Gaetz 1989; Rosner & Tucker 1989; Pistinner & Shaviv 1996; Dos Santos 2001). While the idea is attractive, it requires extremely efficient conduction, which is considered problematic.

In a classic paper, Spitzer (1962) showed that thermal conduction in an unmagnetized plasma has a diffusion constant,[4] $\kappa_{\rm Sp} \sim \lambda^2/t_{\rm Coul} = \lambda v_t \sim 4 \times 10^{32} T_1^{5/2} n_{-3}^{-1}$ cm^2 s^{-1}, where $t_{\rm Coul} = \lambda/v_t$ is the mean free time between Coulomb collisions and λ and v_t are the mean free path and the thermal speed of electrons (Cowie & McKee 1977; Ettori & Fabian 2000): $\lambda \sim 30 T_1^2 n_{-3}^{-1}$ kpc and $v_t \sim (kT/m_e)^{1/2} \sim 4 \times 10^9 T_1^{1/2}$ cm^2 s^{-1}. Here, $T_1 = kT/10$ keV is the scaled temperature, $n_{-3} = n/10^{-3}$ cm^{-3} is the scaled electron number density, and we have used an average value for the Coulomb logarithm, $\ln \Lambda \sim 38$.

The time required for heat to diffuse conductively across a radius $R = 100R_2$ kpc is given by $t_{\rm Sp} \sim R^2/\kappa_{\rm Sp} \sim 8 \times 10^6 T_1^{-5/2} n_{-3} R_2^2$ yr. For conduction to have a significant effect on a cooling flow, the conduction time must be comparable to the cooling time $t_{\rm cool}$ of the gas. Table 1 lists representative data for two clusters, Hydra A (David et al. 2001) and 3C 295 (Allen et al. 2001), at two characteristic radii, 100 and 10 kpc. Columns (6) and (7) give $t_{\rm cool}$ and $t_{\rm Sp}$. We see that if thermal conduction in a cluster is as efficient as in Spitzer's theory, or even if it is a factor of a few less efficient, heat conduction will have a strong effect on the energetics of a cooling flow and perhaps will shut off mass dropout. The main problem with this idea is that the gas in a cluster is likely to be magnetized, and conventional wisdom says that magnetic fields severely suppress conduction relative to the Spitzer level. This is the topic of the present Letter.

We discuss, in § 2.1, the theory of conduction in a tangled magnetic field as developed by Rechester & Rosenbluth (1978, hereafter RR) and Chandran & Cowley (1998, hereafter CC); the theory predicts that the coefficient of thermal conduction is a factor of ∼100–1000 lower than the Spitzer coefficient. We then present, in §§ 2.2 and 2.3, an extension of the theory to a turbulent medium; we show that if turbulence extends over a factor of 100 or more in length scale, thermal conduction is almost as efficient as in Spitzer's theory. We conclude with a brief discussion in § 3.

2. THEORY OF THERMAL CONDUCTION IN A WEAKLY COLLISIONAL MAGNETIZED GAS

2.1. *Conduction in a Chaotic Magnetic Field with a Single Scale*

In the presence of an ordered magnetic field, conduction is anisotropic. Electrons stream freely parallel to the field line, so the parallel diffusion constant is almost equal to the Spitzer value: $\kappa_\parallel \sim \kappa_{\rm Sp}/3$. (The factor of $\frac{1}{3}$ is because diffusion is in one dimension rather than three; see CC.) Perpendicular to the field, however, electrons follow circular Larmor orbits with radius $\rho_e \ll \lambda$. Since an electron moves only a distance $\sim\rho_e$ in each scattering, the perpendicular diffusion constant is given by $\kappa_\perp \sim \rho_e^2/t_{\rm Coul} \sim (\rho_e/\lambda)^2 \kappa_{\rm Sp} \ll \kappa_{\rm Sp}$. For a galaxy cluster with a magnetic field of $\sim 10^{-6}$ G, we have $\rho_e \sim 10^{-12}\lambda$, and $\kappa_\perp$ is effectively zero.

Thermal conduction behaves very differently when the magnetic field is chaotic. The theory for a tangled field with a single coherence length l_B was developed by RR and has been recently revived in the astrophysical context by CC. Since the field is chaotic, the separation r of two nearby field lines must

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[4] The coefficient κ_c, which enters in the heat flux equation $q = -\kappa_c \nabla T$, is related to $\kappa_{\rm Sp}$ by $\kappa_c = n k_{\rm B} \kappa_{\rm Sp}$, where $k_{\rm B}$ is the Boltzmann constant. Note also that heat diffusion and particle diffusion have slightly different coefficients, differing by a factor of order unity.

TABLE 1
Comparison of the Cooling Time and the Conduction Time in Hydra A and 3C 295

Cluster Name (1)	R (kpc) (2)	n (cm^{-3}) (3)	kT (keV) (4)	λ (kpc) (5)	t_{cool} (Gyr) (6)	t_{Sp} (Gyr) (7)	t_{turb} (Gyr) (8)
Hydra A	100	0.005	3.6	0.8	5	0.5	2
	10	0.06	3.1	0.05	0.5	0.09	0.4
3C 295	100	0.008	5.0	0.9	7	0.3	2
	10	0.15	3.0	0.02	0.3	0.2	1

have a Lyapunov-like scaling as a function of distance l along the field:

$$r \sim r_0 \ \exp\,(l/L_{\mathrm{Lyap}}), \tag{1}$$

where r_0 is the initial separation of the two lines. Because there is only one characteristic scale in the problem, namely l_B, we expect $L_{\mathrm{Lyap}} \sim l_B$.

Following RR and CC, let us consider the evolution of a compact cloud of electrons of initial size ρ_e. With time, the electrons diffuse parallel to the field, with a diffusion constant $\kappa_\parallel$. As the electron cloud spreads out, its perpendicular extent diverges exponentially according to equation (1). Thus, by the time the electrons have diffused a Rechester-Rosenbluth distance $L_{\mathrm{RR}} \sim l_B \ln\,(l_B/\rho_e) \sim 30 l_B$ along the field line, their transverse separation is of order l_B. The numerical coefficient 30 corresponds to $l_B/\rho_e \sim 10^{13}$, a typical value for a galaxy cluster (assuming that l_B is a fraction of the radius). Being a logarithmic factor, the numerical value is insensitive to details. When electrons have moved a distance L_{RR} along the tangled field line, their three-dimensional rms displacement is R_*, where $R_*^2 \sim L_{\mathrm{RR}} l_B \sim 30 l_B^2$. Beyond R_*, the motion of an electron is isotropic and uncorrelated with its previous path.

Let us define t_* as the time it takes for electrons to diffuse a distance L_{RR} along the field: $t_* \sim L_{\mathrm{RR}}^2/\kappa_\parallel$. For $t < t_*$, electrons diffuse anisotropically: $l \sim (\kappa_\parallel t)^{1/2}$, $r \sim \rho_e \exp\,[(\kappa_\parallel t)^{1/2}/l_B]$. For $t > t_*$, however, electrons diffuse isotropically and move in three dimensions, according to $R \sim (\kappa_* t)^{1/2}$, where

$$\kappa_* \sim R_*^2/t_* \sim (l_B/L_{\mathrm{RR}})\kappa_\parallel \sim 10^{-2}\kappa_{\mathrm{Sp}}. \tag{2}$$

We see that for $R > R_*$ the conduction is many orders of magnitude more efficient than when the field is ordered. However, κ_* is still a factor of ~100 less than κ_{Sp}. The conduction time is correspondingly ~100 times longer than the Spitzer time t_{Sp}. As Table 1 shows, such weak conduction is unlikely to have an important effect on cooling flows.

The estimate given in equation (2) is valid so long as $\lambda < l_B$. This condition, which is likely to be satisfied by the gas in clusters [compare λ with $l_B \sim R$/(few) in Table 1], ensures that collisions enable electrons to pass through magnetic mirrors caused by inhomogeneities in the field. If $\lambda > l_B$, conduction is suppressed by an additional factor $\theta < 1$ (Chandran et al. 1999; Malyshkin & Kulsrud 2001), since only a fraction of the electrons are able to penetrate the mirrors. This would cause the conduction time to increase by a factor of $1/\theta$.

2.2. *Conduction in a Multiscale Chaotic Magnetic Field*

A key assumption of the RR theory is the presence of a single Lyapunov length scale $L_{\mathrm{Lyap}} \sim l_B$. However, if the medium is turbulent, chaotic fluctuations will be present over a wide range of length scales. We generalize the theory for such a multiscale medium.

We begin by reexpressing the single-scale theory as follows. When two field lines are separated by a distance r smaller than l_B, their mean square separation $\langle r^2\rangle$ increases with distance l along the field line according to a Lyapunov-like scaling. However, when $r > l_B$, the increase is given by the usual diffusion law, where $\langle r^2\rangle$ increases by $\Delta r^2 \sim l_B^2$ for a parallel displacement $\Delta l \sim l_B$. Thus, we may describe the evolution in the two regimes, $\langle r^2\rangle < l_B^2$ and $\langle r^2\rangle > l_B^2$, by the following two differential equations

$$d\langle r^2\rangle/dl \sim 2\langle r^2\rangle/l_B, \quad d\langle r^2\rangle/dl \sim l_B^2/l_B = l_B. \tag{3}$$

Consider now a tangled magnetic field with a range of scales, and assume that the statistics of the magnetic field fluctuations are described by the Goldreich & Sridhar (1995, hereafter GS) theory of Alfvénic MHD turbulence. In a GS turbulent cascade, there is a range of scales $l_\perp$ perpendicular to the field, extending from a minimum scale $l_{\perp,\,\mathrm{min}}$ to a maximum scale l_B. The fluctuations are anisotropic, so that for a given perpendicular scale $l_\perp$ the corresponding parallel coherence scale $l_\parallel$ is given by

$$(l_\parallel/l_B) \sim (l_\perp/l_B)^\alpha, \quad l_{\perp,\,\mathrm{min}} < l_\perp < l_B. \tag{4}$$

For simplicity, we have selected the normalization such that, on the outer scale l_B, the fluctuations are isotropic: $l_\perp \sim l_\parallel \sim l_B$. The index α is equal to $\frac{2}{3}$ for strong MHD turbulence (GS) and $\frac{3}{4}$ for intermediate turbulence (Goldreich & Sridhar 1997).

Using the single-scale equations (eq. [3]) as a guide, it is straightforward to write corresponding equations for a medium with a spectrum of fluctuations. We then identify three regimes for the evolution of $\langle r^2\rangle$.

First, when $r \equiv \langle r^2\rangle^{1/2} < l_{\perp,\,\mathrm{min}}$, all the fluctuation scales in the medium contribute to Lyapunov-like growth (assuming that they all behave chaotically), so

$$\frac{d\langle r^2\rangle}{dl} \sim 2\langle r^2\rangle \int_{1/l_B}^{1/l_{\perp,\,\mathrm{min}}} \frac{d\ln k_\perp}{l_\parallel} \sim \frac{2\langle r^2\rangle}{l_{\parallel,\,\mathrm{min}}}, \tag{5}$$

where we have written $k_\perp = 1/l_\perp$ and ignored constants of order unity in the normalization of the integral. We see that the effective Lyapunov scale for the growth of r is the parallel coherence length of the *smallest*-scale fluctuations in the medium, i.e.,

$$r \sim r_0 \ \exp\,(l/l_{\parallel,\,\mathrm{min}}), \quad l_{\parallel,\,\mathrm{min}} \sim l_{\perp,\,\mathrm{min}}^\alpha l_B^{1-\alpha}. \tag{6}$$

Since usually $l_{\parallel,\,\mathrm{min}} \ll l_B$, the growth is rapid.

Once r exceeds $l_{\perp,\,\mathrm{min}}$, the evolution switches to a second regime. We continue to have Lyapunov-like growth from scales $l_\perp > r$, but there is diffusion-like growth for scales $l_\perp < r$. The evolution equation for $l_{\perp,\,\mathrm{min}}^2 < \langle r^2\rangle < l_B^2$ thus becomes

$$\frac{d\langle r^2\rangle}{dl} \sim 2\langle r^2\rangle \int_{1/l_B}^{1/r} \frac{d\ln k_\perp}{l_\parallel} + \int_{1/r}^{1/l_{\perp,\,\mathrm{min}}} d\ln k_\perp \frac{l_\perp^2}{l_\parallel}. \tag{7}$$

If $l_B \gg r \gg l_{\perp,\,\mathrm{min}}$, each integral in equation (7) is dominated by the scale r. Substituting $l_\parallel$ from equation (4) in differential equation (7), we then obtain the solution

$$r/l_B \sim (l/l_B)^{1/\alpha}. \tag{8}$$

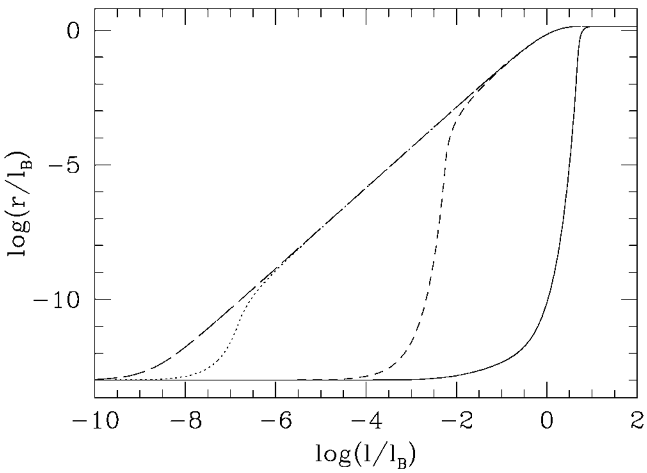

FIG. 1.—Variation of the rms transverse displacement r of two field lines vs. longitudinal distance l along the field lines, for $\alpha = \frac{2}{3}$. Four choices of the minimum scale of turbulence are shown, $l_{\perp,\min}/l_B = 10^{-1}$ (*solid line*), 10^{-5} (*short-dashed line*), 10^{-11} ($\sim\rho_p/l_B$; *dotted line*), and 10^{-13} ($\sim\rho_e/l_B$; *long-dashed line*), which correspond to $l_{\parallel,\min}/l_B = 2.2 \times 10^{-1}$, 4.6×10^{-4}, 4.6×10^{-8}, and 2.2×10^{-9}, respectively.

Remarkably, the separation between two neighboring field lines becomes of order l_B for a parallel translation of only $\sim l_B$; this is much faster than in the RR theory, which requires a parallel translation $\sim 30 l_B$. The solution $r \propto l^{1/\alpha}$ corresponds exactly to $l_\perp \propto l_\parallel^{1/\alpha}$ in the turbulence model (eq. [4]). Thus, the rms separation of field lines grows along the "Goldreich-Sridhar cone."

When $r > l_B$, we enter a third regime, which corresponds to isotropic diffusion. From equation (8), it is clear that thermal conduction in a multiscale chaotic field is almost as efficient as in Spitzer's theory. Replacing $L_{\rm RR}$ with l_B in equation (2), we estimate $\kappa_{\rm turb} \sim \kappa_{\rm Sp}/3$ (but see § 2.3 for a better estimate of the coefficient). As in § 2.1, we have assumed $\lambda < l_B$ and have not included a magnetic mirror factor θ. In GS turbulence, perturbations on length scales $l_\perp < l_B$ have weak magnetic field fluctuations, $\Delta B/B \sim l_\perp/l_\parallel \sim (l_\perp/l_B)^{1-\alpha} < 1$, and cause negligible mirroring. Only perturbations on the scale l_B cause strong mirroring, but these have a negligible effect so long as $\lambda < l_B$ (Chandran et al. 1999; Malyshkin & Kulsrud 2001).

2.3. *Numerical Solutions*

We have numerically integrated the differential equations (5) and (7), starting with an initial separation $r = 10^{-13} l_B \sim \rho_e$, and assuming $\alpha = \frac{2}{3}$ as appropriate for strong turbulence in the GS model. Figure 1 shows four numerical solutions for the evolution of r as a function of distance l along the field line, corresponding to four choices of the minimum scale $l_{\perp,\min}$ of the turbulence. We see exponential growth of r for $l < l_{\perp,\min}$ and power-law growth for larger separations, confirming the scalings given in equations (6) and (8).

Let us define the decorrelation length $L_{\rm dec}$ as the distance along the field line for which the transverse separation r becomes equal to l_B. Figure 2 shows how $L_{\rm dec}$ depends on $l_{\perp,\min}$. When $l_{\perp,\min} \sim l_B$, the turbulence is dominated by a single (outer) scale. This corresponds to the RR theory, and in this limit the decorrelation length is large, as expected. However, for $l_{\perp,\min} \lesssim 10^{-2} l_B$, we find that $L_{\rm dec}$ is quite small, asymptoting to $\sim 1.6 l_B$. Since $L_{\rm dec}$ is the analog of the Rechester-Rosenbluth length $L_{\rm RR}$ for a multiscale medium, we may replace $L_{\rm RR}$ with $L_{\rm dec}$ in equation (2) to estimate the diffusion constant in a turbulent medium:[5]

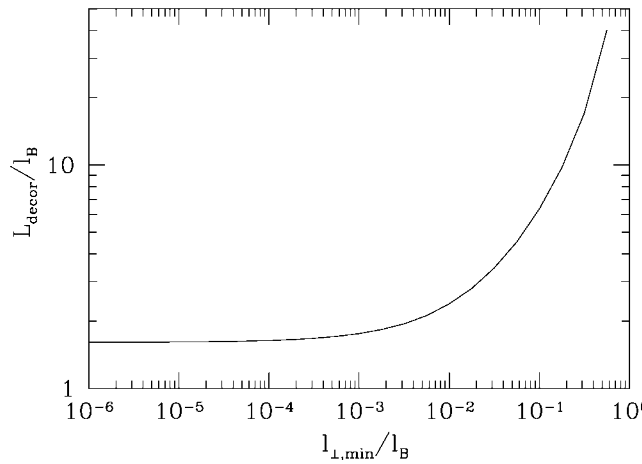

FIG. 2.—Variation of the decorrelation length $L_{\rm dec}$ vs. $l_{\perp,\min}$. Note that $L_{\rm dec} \lesssim 2 l_B$ for $l_{\perp,\min} \lesssim 10^{-2} l_B$.

$$\kappa_{\rm turb} \sim (l_B/L_{\rm dec})\kappa_\parallel \sim \kappa_{\rm Sp}/5, \qquad t_{\rm turb} \sim 5 t_{\rm Sp}. \tag{9}$$

Thus, if turbulence extends over at least two decades in scale, conduction is very efficient and approaches the Spitzer level to within a factor of a few.

3. DISCUSSION

Thermal conductivity in a homogeneous magnetic field is known to be highly anisotropic—it is Spitzer along the field, but extraordinarily reduced in the transverse direction. RR came up with the important insight that when the magnetic field is tangled and chaotic, thermal conduction is enhanced significantly by the exponential divergence of neighboring field lines. However, even with this effect, CC estimated that the conductivity in galaxy clusters is below the Spitzer level by a factor $\gtrsim 100$.

We have shown in this Letter that if the field is chaotic over a wide range of length scales (factor of 100 or more), as might happen with MHD turbulence (GS), thermal conduction is boosted to within a factor of ~ 5 of the Spitzer value. Such strong conduction will have a significant effect on galaxy clusters (compare cols. [6] and [8] in Table 1). It can transport heat to the center of a cluster to replace the energy lost through cooling, and it can also eliminate any thermal instability in the cooling gas. Thus, it may well reduce the need for large-scale mass dropout in cooling flows (Tucker & Rosner 1983; Bertschinger & Meiksin 1986). Only in the inner regions of some clusters (e.g., 3C 295; see Table 1) might there be significant dropout. It is worth noting that some authors have discussed potential problems with invoking such strong conduction (Binney & Cowie 1981; Bregman & David 1988), while others have rebutted these arguments (Rosner & Tucker 1989; Dos Santos 2001).

An important requirement for the validity of our analysis is that the magnetic field should behave chaotically; i.e., it should exhibit Lyapunov-like behavior over a wide range of scales. Weak MHD turbulence consists of a superposition of Alfvén waves and is not chaotic. However, the model of strong and

[5] Since eqs. (5) and (7) are approximate and may contain numerical coefficients of order unity multiplying the integrals, the numerical factor of $\frac{1}{5}$ in eq. (9) is approximate as well.

intermediate MHD turbulence developed by GS is chaotic, as indicated by the breakdown of perturbation theory (Goldreich & Sridhar 1997).

It should be noted that our theory of thermal conduction does not require ongoing dynamic turbulence. Each episode of dynamic turbulence will leave behind a substantial level of frozen-in tangled fields, even after the turbulent motions have ceased. Such leftover tangled fields should be sufficient to enhance conduction to the levels we estimate.

As a final point, we should discuss a serious caveat. *Chandra* has found evidence of sharp temperature jumps in a few clusters, e.g., Abell 2142 (Markevitch et al. 2000) and Abell 3667 (Vikhlinin, Markevitch, & Murray 2001). The observations indicate that conduction across the temperature jumps is far below the Spitzer level (Markevitch et al. 2000; Ettori & Fabian 2000), in apparent contradiction with the estimate presented here. Vikhlinin et al. (2001) propose that the magnetic field is stretched parallel to the interface, thus inhibiting diffusion across the field on small scales. Extending this idea, we suggest that when two distinct objects merge, as seems to be the case with the above clusters, the two regions (each of which is internally chaotic and highly conducting) may be thermally isolated from each other because their magnetic fields have not yet interpenetrated each other. It is unclear how long such magnetic isolation will survive.

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REFERENCES

Allen, S. W., et al. 2001, MNRAS, 324, 842
Bertschinger, E., & Meiksin, A. 1986, ApJ, 306, L1
Binney, J., & Cowie, L. L. 1981, ApJ, 247, 464
Böhringer, H., et al. 2001, A&A, 365, L181
Bregman, J. N., & David, L. P. 1988, ApJ, 326, 639
Chandran, B. D. G., & Cowley, S. C. 1998, Phys. Rev. Lett., 80, 3077 (CC)
Chandran, B. D. G., Cowley, S. C., Ivanushkina, M., & Sydora, R. 1999, ApJ, 525, 638
Cowie, L. L., & McKee, C. F. 1977, ApJ, 211, 135
David, L. P., Nulsen, P. E. J., McNamara, B. R., Forman, W., Jones, C., Ponman, T., Robertson, B., & Wise, M. 2001, ApJ, 557, 546
Dos Santos, S. 2001, MNRAS, 323, 930
Ettori, S., & Fabian, A. C. 2000, MNRAS, 317, L57
Fabian, A. C. 1994, ARA&A, 32, 277
Fabian, A. C., Mushotzky, R. F., Nulsen, P. E. J., & Peterson, J. R. 2001, MNRAS, 321, L20
Gaetz, T. J. 1989, ApJ, 345, 666
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763 (GS)
———. 1997, ApJ, 485, 680
Malyshkin, L., & Kulsrud, R. 2001, ApJ, 549, 402
Markevitch, M., et al. 2000, ApJ, 541, 542
Molendi, S., & Pizzolato, F. 2001, ApJ, 560, 194
Pistinner, S., & Shaviv, G. 1996, ApJ, 459, 147
Rechester, A. B., & Rosenbluth, M. N. 1978, Phys. Rev. Lett., 40, 38 (RR)
Rosner, R., & Tucker, W. H. 1989, ApJ, 338, 761
Sarazin, C. L. 1988, X-Ray Emission from Clusters of Galaxies (Cambridge: Cambridge Univ. Press)
Spitzer, L., Jr. 1962, Physics of Fully Ionized Gases (2d ed.; New York: Interscience)
Tucker, W. H., & Rosner, R. 1983, ApJ, 267, 547
Vikhlinin, A., Markevitch, M., & Murray, S. S. 2001, ApJ, 551, 160